Gadanidis, G., Gadanidis, M.J. & Huang, A.H. (2005). Using Humor to Gain Mathematical Insight. *Mathematics Teaching in the Middle School 10*(5), 244-250. [pre-publication draft]

Using Humor to Gain Mathematical Insight

Sometimes missing in mathematics classrooms is mathematical humor, which might alleviate the tension that many students experience. Also needed are opportunities for mathematical insight, through which students might sense the pleasure of doing mathematics. Humor and insight have many similarities. Both are creative acts. Both emerge from our predisposition to explore and take pleasure in novel ways of seeing and understanding our world. We are drawn to attend to apparent dissonance, to seek to understand complexity, to make ourselves more complex, and this feels good.

In this paper we explore humor and insight in teaching and learning mathematics. First we discuss mathematical humor and insight in isolation. Then we consider the conjunction of mathematical humor and insight at the classroom level.

Humor

Boyd (1999) suggests that we might be biologically predisposed for the pleasure resulting from a humorous moment, the pleasure associated with "attending to incongruity."

> Our pleasure in laughter … evolved as a reward for consciously attending to the disparity between anticipation and actualization, for alertness to incongruity, for openness to situations where we might need to adjust expectations rapidly, in short for flexibility of response. (p. 319)

We respond to humor immediately, "we 'get' it at once … or not at all" (Boyd 1999, p. 315). "*Shared* humor is an invitation to inclusion … to reaffirm our friendly intent" (p. 319).

Please turn the volume down; it's getting too big in here!

The above statement initially confuses. We are familiar with complaints about music that is played too loudly. However, the second part of the statement switches the meaning of volume from the measurement of sound to the measurement of space. We attend to the incongruity, we recognize the pun, and perhaps we smile or laugh. Not all incongruous statements or situations are funny. There has to be some connection between the two parts of the statement to make it humorous. For example, "Please turn the volume down; it's raining in here!" would not have a similarly humorous effect.

There are several benefits to using humor in the mathematics classroom (Martin & Baksh 1995; Cornett 1986, 2001; Dyer 1997; Medgyes 2002; Wischnewski 1986).

- Humor helps create a more positive learning environment. It helps reduce barriers to communication and increase rapport between teacher and students.
- Humor helps gain student attention and keep their interest in classroom activity.
- By reducing stress and anxiety, humor helps improve comprehension and cognitive retention.
- Humor improves student attitude towards the subject.
- Humor helps communicate to students that it is okay for them to be creative, to take chances, look at things in an offbeat way, and, perhaps, even make mistakes in the process.
- Humor can help students see concepts in new light, and increase their understanding.

- The use of humor is rewarding for the teacher, knowing that students are listening with enjoyment.

Insight

If humor is instant gratification – we get and enjoy the joke right away – then insight is delayed gratification. Though insightful moments do often occur in an instant, where we suddenly see something that was not apparent to us before, they are usually preceded by prolonged periods of effort and attention.

Aha moments are inspiring. They are moments of deeper understanding, they have the power to transform attitudes and beliefs about what is mathematics and they inspire students to seek more of these moments (Liljedahl 2002). Moments of significant insight fix the experience in one's consciousness, enriched by a strong sense of accomplishment and confidence (Burton 1984). Insight leads to a feeling of satisfaction, a sense of great clarity, and is accompanied by a positive emotional response. There is a sense of 'authorship' of the ideas, which provides an important element of self-affirmation and confidence (Barnes 2000).

As is also the case with humor, we are predisposed to attend to incongruity, to seek to resolve it, and this giving of attention feels good. Davey (1999) argues that attention is aesthetic in nature. Whenever we bring consciousness to bear upon a topic, either individually or communally, we engage it in emotional and imaginative ways. We use attention to gain insight, to learn and extend ourselves, to incorporate a new thing, whether that is a new way of solving an old problem or finding new ways to express an idea. We extend our understanding, we become more complex, and this feels good. In Dissanayake's (1992) words, "as *Homo aestheticus*, we really require beauty and meaning." Attention is our way of gaining beauty and meaning from experience.

If we see the mathematics classroom experience unfolding 'story-like' (Gadanidis and Hoogland 2003), then the good mathematics story offers students the incentive to give their attention and the opportunity to gain mathematical insight. McKee (1997, p. 237), writing about what constitutes a good story, states: "Insight is the audience's reward for paying attention, and a beautifully designed story delivers this pleasure scene after scene after scene." Good stories are often about things that are not routine-like and involve some element of surprise, as is also the case with humor. Good stories trigger pleasurable reactions as they offer us opportunities "to use our minds in fresh, experimental ways, to flex our emotions, to enjoy, to learn, to add depth to our days" (McKee 1997, p. 5). Setting, rising action, with some sort of 'conflict', and the resolution of conflict are integral components of good stories. Good stories represented as graphs would have peaks and valleys. The typical mathematics experience "authored" for students makes for a weak "story" as it resorts to substituting information for insight (McKee 1997; Schank 1990). By relying on shortcuts to mathematical insight, classroom mathematics becomes more like "the pamphlet version of discovery with numbered steps" where in the rush to conclusions and rules, students miss the pleasure of the process, of the journey (Gillard 1996, p. xi).

Teachers, like writers, are "moved by a desire to touch the audience" (McKee 1997, p.7) and seek "to be part of a child's discovery" (Gadanidis et al 2003; Gadanidis and Hoogland 2003). For this to be possible there needs to be a focus on investigation of mathematical relationships. For example, the typical approach to area and perimeter in the middle grades is reliant on the development and use of formulas, with minimal emphasis on relationships. As one teacher commented, "I think I do more the traditional way, separating the two." Another said, "I do perimeter first" (Gadanidis 2001, p. 229). However, when area and perimeter are related in an area optimization problem, such as the one below, students are offered opportunities to notice relationships, to make hypotheses, to imagine alternatives, and to gain mathematical insight.

Consider the example:

> We're going to build a pen for your dog. Your parents are going to go out and buy 24 meters of fence. You're going to build it with four sides. How are you going to get the biggest area for your dog to play?

One teacher reflected on the effect on students: "When I introduced area and perimeter [in previous years] they didn't see a relationship between the two, but when you fix one at a given constant, and you can manipulate the other, they really find that pretty neat" (p. 229). Teachers were pleased to see that "all of them got it" and that some students asked "is the biggest area always a square? … wouldn't a circle be bigger? … they were really thinking" (p. 228).

Moments of insight are pleasurable for students, as they sense mathematical connections and develop an appreciation of mathematical structures and relationships. As one elementary pre-service teacher commented, "I love feeling my brain tumble over as it understands something for the first time … The moments that I've experienced in math when the lights have gone on and I felt like "I got it" were in fact beautiful moments, which I clearly remember even to this day" (Gadanidis et al submitted). Moments of student insight are also pleasurable for teachers. Such pleasure results from (1) vicariously (re)experiencing students' aha moments and (2) from authoring a good mathematics story and seeing students respond as they had hoped they would when they planned their lesson (Boyd 1999).

From haha to aha in the mathematics classroom

By the middle grades, many students develop a view of mathematics as learning procedures and getting correct answers. They are not used to attending deeply to mathematical relationships and seeking the pleasure of mathematical insight. Humor may be a way of cleansing students' mathematical palettes, a way of engaging students creatively with mathematics and setting the stage for attending deeply to mathematics (Gadanidis 1999). Humor may be the opening act of a classroom story that offers students the opportunity to take pleasure in attending to mathematics and gaining new insight. Below we share some examples of opportunities for middle grade students to experience mathematical humor and insight.

Jokes and comics. The comic in Figure 1, with its pun "breakfast at (7,15)," may be an introduction to an investigation of ordered pairs and their relationships. The ordered pair (7,15) at first surprises. However, we can see the spider web as a grid, the muntin (the strips separating the panes of glass in a window sash) as a set of axes, the coordinates of the fly in the web, and we know that spiders do eat flies (sometimes for breakfast). Of course, some mathematical background and some creativity are required to make these connections.

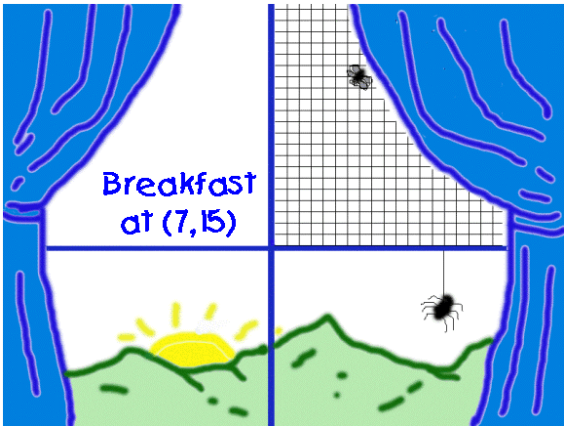

Figure 1. A mathematical comic.

The "breakfast at (7,15)" comic may be followed by an investigation of relationships among ordered pairs. For example, students may be asked to identify the path the spider would follow if it is limited to "stepping" on ordered pairs whose coordinates have a sum of 10. This is an informal way of exploring the linear function $x + y = 10$ and may lead to an investigation of other, similar linear functions. Some other questions that may be considered: What would be a coordinate rule that would lead to the fly? Is there more than one rule

that would work? These questions, in the context of the comic in Figure 1, offer a playful investigation of the family of linear functions that pass through (7,15). Such investigation offers students the opportunity to gain insight into relationships among ordered pair patterns, graphs and equations in the context of linear functions and anchors their understanding to meaningful, imaginative and interesting experiences.

Poems. Exploring mathematical similes and metaphors may be a first step toward mathematical poetry (Gadanidis 1999). The poem in Figure 2 (Kong 1999) was written in a class where students created and explained similes and metaphors on mathematics topics of their choice. Then they used the ideas that emerged to write poems. Such student work may be easily integrated with language arts studies and may be displayed on a school bulletin board or Web site.

Numeration

Numeration is a murderer,
7 ate 9,
When he got into court,
He said, 'I had to dine!'

Figure 2. A mathematical poem.

Interviews. Interviews with mathematics concepts, ideas or historical figures are excellent ways of getting to the essential attributes of concepts. One such example is the interview with the "circle," shown in Figure 3 (Gadanidis 1999). Students have tremendous fun authoring such interviews and acting them out for their peers. Prior to the authoring of interviews, we ask students working in small groups to create concept maps that list everything they know and can find out about their topics. Then they identify the essential attributes of the concept and use these as the basis for the interview. The interviews can take a variety of forms, such as a talk show, a news report, or an infomercial. It is worth the time to have students perform a first draft to another group and seek feedback. Students we have worked with have been eager to add creative touches to their interviews, such as mathematical commercial breaks and breaking mathematics news bulletins. These venues give students who might not necessarily excel in mathematics, another way in.

Interview with Circle

Circle, you've been described as a well-rounded shape. How do you feel about this?

Being well-rounded is the essence of my being.

Does it bother you that you don't have any vertices?

Not at all. I do appreciate vertices, as well as edges. However, not having any does have advantages. Can you imagine a car with square or hexagonal tires?

Figure 3. A mathematical interview.

Skits. Skits may be teacher-prepared or student authored. The skit in Figure 4 (Gadanidis 1995) may be posed as a problem for students to solve. The skit takes place in Fractionville, where common denominators determine who your friends will be. Students may be asked to work in small groups to discuss the problem and author a "happy ending" for the story (which might involve the fractions changing their denominators without changing their value). Skits lend themselves nicely to a variety of mathematical topics. Some examples are listed below:

- Why two wrongs don't make a right but two negatives make a positive (when multiplying). A sample integers skit is shown in Figure 5 (Gadanidis 1995).
- But doctor, I'm supposed to be irrational! (π is an irrational number that middle school students become familiar with, in the context of circle area and circumference)

- The real square (and why does it think it's a rectangle?)
- Taking chances (with probability).

We have noticed that students of all ages enjoy authoring and performing mathematical skits. In one of our projects, secondary school drama students authored or adapted such skits and performed them for elementary school students. As starting points, the drama students were given some teacher-authored skits, like those in Figures 4 and 5.

Ads. The writing of mathematical ads may be used as a warm-up activity prior to authoring interviews or skits. They are often fun and, because of their economy of words, focus student attention on the essential attributes of mathematical concepts. A sample ad is shown in Figure 6 (Gadanidis 2000).

Co-ordinate yourself!
Used axes to the highest bidder. Build a co-ordinate system or add a new dimension. The possibilities are infinite. Auction Tuesday at (8,10).

Figure 6. A mathematical ad.

Conclusion

Mathematical humor and insight are creative and pleasurable experiences. Both develop a positive attitude towards mathematics and facilitate higher order thinking. Our classroom experience suggests that many middle school students have come to view mathematics as a superficial and an uncreative activity, where they learn procedures and solve predictable problems. At the same time, we have found that given the opportunity students are eager to think mathematically at higher levels, to make connections and conjectures, and to extend investigations but posing their own questions.

Mathematical humor may serve as a steppingstone for students – and teachers – to attend deeply and creatively to mathematics and experience the pleasure of mathematical insight. Teachers looking for ideas would find helpful information by doing a Web search with such key terms as *math comics*, *math humor* and *math poetry*. Care needs to be taken that humor does not simply serve the purpose of sugarcoating for superficial mathematics, but rather becomes a motivation for attending to big mathematical ideas, which integrate and link mathematical concepts in complex and meaningful ways.

Can I be your friend?
ONE-THIRD: I'm sorry One-half, but we're not compatible.
ONE-HALF: What do you mean?
ONE-THIRD: Your denominator is two and mine is three. We need to have the same denominator to be friends.
ONE-HALF: That's silly! Who made up that rule. Look, our numerators are the same. One-quarter's numerator is also one. Why can't we all be friends?
ONE-QUARTER: When it comes to friendship among fractions, One-half, it's the denominator that matters.
ONE-THIRD: Yes. It's the denominator that matters in friendship.
ONE-HALF: I'll be a good friend to you.
ONE-QUARTER: I'm sorry, One-half. It's just how things are in Fractionville. You're new in town, but you'll get used to it eventually.
ONE-THIRD: Yes, you'll get used to it. In any case, One-half, there are many other fractions that you can try to be friends with, like Three-halves or Five-halves.
ONE-HALF: But aren't those *improper* fractions? My mom always warned me to stay away from them.
ONE-QUARTER: Now look who's making up silly rules about friendship.
ONE-THIRD: Yes, One-half, I have a lot of fraction friends that are improper and I like them very much.
ONE-HALF: Well if I can be friends with an improper fraction, surely I can be friends with you two! Why don't we try to find a way?

Figure 4. A fractions skit (unfinished).

ALEXANDER: Hi Mom.
MOM: Hi honey. How was school today?
ALEXANDER: School was great Mom. We had an excellent math class. We learned about integers.
MOM: But then what's wrong honey? You look so sad.
ALEXANDER: Look at my phone bill, Mom! It's thirty-five dollars. Maybe getting my own phone wasn't such a good idea. I have a very negative feeling about it.
MOM: I have a bit of extra money this month. Why don't I take it and pay it for you?
ALEXANDER: Oh Mom! You really mean it?
MOM: Yes, honey. But just this time, okay? You will have to be more careful next month.
ALEXANDER: Thanks, Mom. You know, I don't feel so negative about the phone bill any more. In fact, I feel quite positive right now. We talked about this in math class today.
MOM: What do you mean, Alexander?
ALEXANDER: It's like integers, Mom. A phone bill is negative in that you have to pay for it. And having something taken away is also negative, because you lose something. But if you put the two negatives together, you get a positive.
MOM: What you're saying is that I took a negative from you so you feel positive.
ALEXANDER: Yes, that's it Mom.
MOM: Well, to tell you the truth dear, I'm feeling a little negative about your phone bill now.
ALEXANDER: Sorry, Mom. It's just how integers work.

Figure 5. An integers skit.

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