CHORUS, COLOUR AND CONTRARINESS IN SCHOOL MATHEMATICS

Nathalie Sinclair
Michigan State University

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I'd like to begin with a quotation from the literature.

> *[he] is plunged into a kind of idleness – he is intransitive; he is, in short, serious: instead of functioning himself, instead of gaining access to the magic [...], to the pleasure [...], he is left with no more than the poor freedom either to accept or reject the text. The text presents itself as transparent [...]. The text present itself in a classic, familiar form. The text is linear, homogeneous, and adheres to the status quo.*

The description above sounds like the typical student's experience of a mathematics text, or perhaps a school mathematics lesson, does it not? The critique is familiar: the ideas in these mathematics texts are presented clearly, in an orderly way, ... but almost monotonously so. This logic is so clear and forceful that the authoritative, linear form beats a repetitive drum: example, exercises; or definitions, deductions, derivations. The subject matter is presented as finished theory, where all is calm... and certain.

But this was not a description of a high school algebra textbook. It's a description of the texts of Dickens, Zola, Tolstoy! The voice is that of the French social and literary critic Roland Barthes; the description, one of what most people consider the great novels, and the paradigms, of imagination and magic. In his book *The Pleasure of Text*, Barthes was trying to distinguish these traditional 19th century literary works, which he called "readerly" texts, from those—to him, contemporary—twentieth century works, like the Nouveau Roman[1] which he called "writerly". Readerly texts, with their dependence on an omniscient narrator and adherence to the unities of time and place, create an illusion of order and significance that is at odds with the chaotic, often directionless, and haphazard nature of modern life. These texts deliver the

1 The Nouveau Roman refers to a movement in French literature of the mid-fifties and early sixties, by authors such as Marguerite Duras and Alain Robbe-Grillet, which called into question the traditional modes of literary realism. It may be seen as a bridge between modernism and postmodernism, the new novel possesses an austere narrative tone that favours a heightened sense of ambiguity with regards to point of view, radical disjunctions of time and space, and self-reflexive commentary on the processes of literary composition.

receiver a fixed, pre-determined reading. In contrast, Barthes saw writerly texts as forcing the reader to produce a meaning or set of meanings that are inevitably other than final or “authorized”—they are personal and provisional, not universal and absolute. Where the readerly text presents itself as a transparent window onto "reality," the writerly text self-consciously acknowledges its artifice by calling attention to the various rhetorical techniques that produce the illusion of realism.

Barthes uses these distinctions to further his own aesthetic and political projects of championing those texts that challenge traditional literary conventions. His main purpose in drawing up these distinctions is to answer the question “How do we enjoy texts?” To be enjoyable, Barthes says, the text must “make the reader no longer a consumer, but a producer," and this led his famous but quixotic pronouncement of the “death of the Author.” Now—at this point—it’s unclear that Barthes’ distinction is valid—and I certainly haven’t described it to the degree that explains why it’s been so significant to post-structural literary theory. But, it does draw our attention to the distinction between thinking of reading as passive consumption of text or as active production of meaning—and it hints at text’s complicity in that distinction. So I would like to propose that if we recognized in Barthes’ description of the readerly text our common critique as educators of the large mass of uninspired texts in school mathematics that it is natural to wonder whether there is some kind of symmetry, namely, whether there also exist writerly mathematical texts. It is also natural to wonder how well this comparison holds up. Certainly, many scholars have shared Barthes’ conception and portrayed mathematics as presenting itself as a transparent window onto reality, and of acting as a singular system that stops or freezes the world, and that—in Barthes’ words— “reduces the plurality of entrances to the world”. And educators’ pleas for multiple representations, multiple solutions, discovery learning, etc, can all be seen as efforts to making mathematics texts a little more writerly, if we broaden our sense of what constitutes a *text* in mathematics. But I believe there’s more to the notion of “writerly” that can provoke our thoughts on the role of story in mathematics and mathematics education.

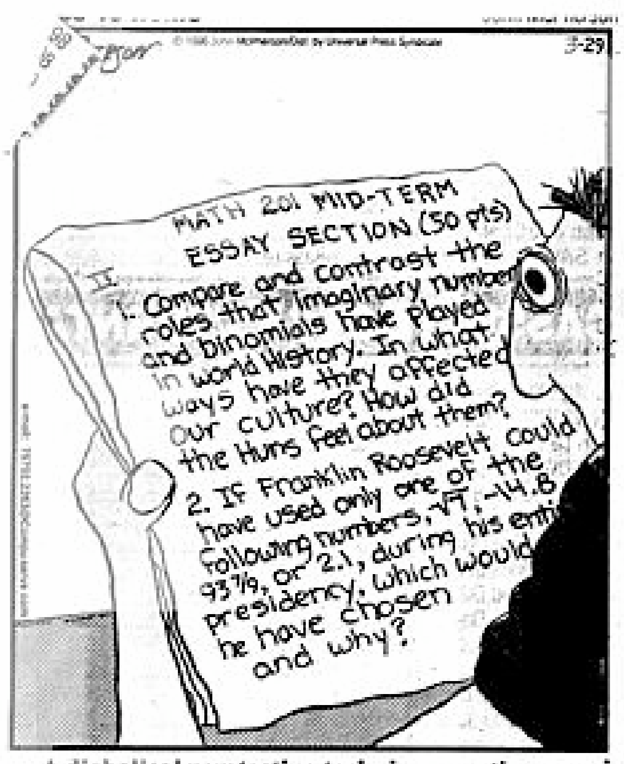
One provocation involves the “linearity” associated with readerly texts—nothing is more linear than mathematical explanation! “Linear” is an adjective *maybe* you can apply to fiction, but it’s the very soul of deductive exposition. So how can we experiment with linearity? Back in fiction, George Landow (1992), an eager adopter of Barthes’ distinctions, writes extensively on the way hypertext can change people’s experiences of fiction. In hypertext, the author suggests the links and the reader navigates through them. Landow seized on the reader's possibility of using hyperlinks to break down the hierarchies and linearities that are imposed in printed text by choosing his or her own way of pursuing the story. So hypertextuality is Landow's vehicle for moving from a readerly, to a writerly, subject position.

Landow saw readers of hypertext choosing their own "center of investigation and experience," and avoiding being "locked into any kind of particular organization or hierarchy." And this is what I tried to do for the student when I experimented with hypertext in a mathematics context, where the text was a mathematical situation. The students were supposed to choose their own paths through the problem through different avenues of problem-posing. And, I hoped that the deliberate decision to choose a path would help students become "active readers," that is, active and aware of the way in which a posed problem creates a fork in the process of inquiry, and leads to different experiences and findings.

The student begins on a page where they are asked to find location on a grid where they might be able to meet Lulu. As they move on the grid, with the help of an arrow, Lulu responds in a particular way, which makes it impossible, for example, for the meeting to occur at the park. But another meeting place is indeed possible, and after finding it—or possibly not—students can find different paths to take, each proposed in the form of a question at the bottom of the page: What if you could start at a different place on the grid? What if you could start at random locations on the grid? Each of these pages changes the conditions of the original one slightly, and engages the student in a different mathematical inquiry. There are 8 pages in total, and while there's a developmental intent—the hardest mathematical ideas aren't found right at the start—the pages can be accessed nonlinearly and possibilities unfold without hierarchy. Eventually all the paths converge at a final screen in which the student can create her own rules for the movements of Lulu—a very writerly activity, for the reader.

The results were very positive, in terms of student engagement and learning, but, in retrospect, more because of the content of each individual page than because of any student "writerly" activities. I found that even hypertext can be prescriptive—so the choice of multiple pathways through hypertextual space may provide only an illusion of emancipation. We replace a fixed outcome with two fixed outcomes. Or three. Other scholars have gone further and suggested that hypertext promotes a channel-flipping mentality rather than a greater, self-critical engagement in the act of reading. But if we take this concern all the way back to Barthes, we find him developing the picture that our experiences of text depend not only on what we read, but on *how we read.*

I thus find it more fruitful to ask not whether mathematical texts can become more writerly, but whether students, and other consumers of mathematics, can become more writerly in their readings of mathematics. In a sense, I am suggesting that instead of creating the texts or stories for students, as I did with Lulu, that we help them become better readers/writers of mathematical stories. Now I've substituted the word "text" for "story," a move I am not quite comfortable making, since it is not at all clear that all texts tell stories. In fact, story-telling depends on the text as well as the audience, the reader/writer, as well as the teller (is a story still a story when it is separated from its telling?). I am also not comfortable about substituting text with story since I find it challenging, in mathematics, to identify just what a story is. Must it have a beginning, middle and end? Must it have setting and plot and characters? What kinds of genres are there? (Both Susan Gerofsky (2003) and Candia Morgan (2003) have investigated some of the genres of stories in mathematics classrooms, including "word problems" and "write-ups," and we have all heard the colourful stories told about mathematicians that serve to pique interest and develop our sense of mathematics' human community: those crazy Pythagorean cultists; Galois and his late-night treatise before death by duel, at dawn; Ramanujan and his impossibly brilliant mental calculating capacity.)



A diabolical new testing technique: math essay questions.

In fact, in addition to the parallels that some have found between story and mathematics, there seem to be many antithetical relationships between the values of the story-teller and those of the mathematician. The cartoon above points to some of the dangers of borrowing form and techniques from other disciplines: and so I ask what tensions might arise when bringing stories in mathematics? Consider, for example, the valuing of embellishment, repetition, exaggeration, ambiguity, plurality, shifting point of view and contradiction in story-tellers, on the one hand, and on the other, the valuing of succinctness, brevity, understatement and coherence in mathematicians. Consider also the importance of context and time in good stories and the absence of them in good mathematics. Though “times” may be changing: Nick Jackiw has pointed out that dynamic geometry software has in fact reintroduced time into mathematics, as one can create and observe continuous transformations of mathematical entities. This aspect of student interaction with dynamic geometry may in fact be more important than we think in terms of allowing students to tell stories about their mathematics. Like authors who manipulate time for narrative effects—a split-second emotional shift may take a chapter to describe, or “years passed” in only two words—so dynamic geometry lets us as educators expand or contract time, unfold it or reinscribe it where it is absent, for pedagogic purposes.

Consider a graph of a function on a regular Cartesian coordinate system.

Now consider this tilted and time-infused version of the Cartesian coordinate system—a dynagraph. As I drag the input along a horizontal axis, the output responds by moving according to a certain rule or function. I see that function being traced out over time, and feel the tempo of it as I drag the input continuously along the axis. You might find yourselves trying to tell a story of how the input affects the output, or, more importantly, how the output *behaves*, always running away from the input in this linear situation (using $y = 2x$), but bopping about a little awkwardly in this discontinuous one (using $y = [x]$). Most people, whether teachers, students or mathematicians, can’t help but giggle at the bopping function, and hearing their sounds may incite you to wonder when the last time was you hear people laugh at a mathematical function. But, it’s perhaps not so surprising. This function has personality. We can watch it with the same curiosity we feel when we watch fellow humans behave… what will they do next? Why are they moving like that? We are much better, as humans, at observing, predicting and explaining behaviour than we are at deducing properties from definitions! Time, motion and image have been inserted into this mathematics, and behaviours, personalities and stories emerge.

I have been focusing on the parallels between mathematics and fiction, but, upon reflection, mathematics seems to map more easily onto poetry— a fact many mathematicians have noted. They can even have a similar visual form! I wonder whether there is a category in mathematics equivalent to the rhyming poem, or the sonnet, poems whose forms define the essence of poetry for many. Perhaps we could compare the sonnet to the proof by contradiction. They may not share constraints in terms of number of lines, but they do share an important convention of form. The Italian sonnet is divided into two parts: the octave and the sestet. As Charles Gayley notes: "The octave bears the burden; a doubt, a problem, a reflection, a query, an historical statement, a cry of indignation or desire [...].” On the other hand, “the sestet eases the load, resolves the problem or doubt, answers the query, solaces the yearning, realizes the vision." Similarly, in the proof by contradiction, the first part poses the doubt while the second part appeases the contrary, burdensome problem.

But here again, in comparing the poem to the proof, a noticeable difference emerges: the byline. Poems are by individuals, and proofs rarely are. (Notice I’ve now taken proofs as the “texts.”) We talk about Pythagorean’s proof of the infinitude of primes or Wiles’ proof of Fermat’s Last Theorem, but most mathematical proofs are anonymous, and for a good reason: they are supposed to exist without their authors or discovers. Where does that leave the reader or the writer of mathematical proofs?

I have mentioned texts, stories and poetry in the context of thinking about mathematics. But when I think of writerly reading, I think of the word *narrative*, and, more precisely, the “narrator” of Ancient Greek theatre. The word originally identifies the chorus, which could have many members, and its role was to provide commentary and questions, give opinions and warnings, and clarify experiences and feelings of the characters in everyday terms. The chorus sympathized with victims, reinforced facts, separated episodes, and often served as spokespeople who reaffirm the status quo. The narrator played the important role of bridging the gap between the audience and the players, to intensify the emotions, and to unite the various episodes—the narrator reads the text in a writerly way.

So now, you will certainly be wondering: wouldn’t a narrator be great in the mathematics classroom. Where students are as mystified by texts as you may be by this Italian folktale:

> *A king fell ill and was told by his doctors, “Majesty, if you want to get well, you’ll have to obtain one of the ogre’s feathers. That will not be easy, since the ogre eats every human he sees.”*

The king passed the word on to everybody, but no one was willing to do to the ogre. Then he asked one of his most loyal and courageous attendants, who said, "I will go."

The man was shown the road and told, "On a mountaintop are seven caves, in one of which lives the ogre."

You will notice, as did Italo Calvino, that "not a word is said about what illness the king was suffering from, or why on earth an ogre should have feathers, or what those caves were like". Indeed, events seem to occur out of nowhere, the seven caves pulled out of a hat like a rabbit; the reader looks for the missing sentence, tries to read in between the lines for missing explanations. Calvino praises the story for its quickness, perhaps as mathematicians would praise proofs for their parsimony. But Umberto Eco (1994) argues for more lingering narrative, one that doesn't make the reader, the student assume there is glue that holds it together, and then feel badly about not finding it. So what could be a lingering narrative in mathematics?

Let's now consider a classic, quick mathematical text: the proof of the irrationality of $\sqrt{2}$. I would ask you to read the un-narrated version first, and to reflect on how it situates you as a participant creating the text.

Suppose $\sqrt{2}$ is not irrational
Then $\exists$ p, q $\in \mathbf{N}$ such that $p/q = \sqrt{2}$ and $(p,q) = 1$
So $p^2 = 2q^2$,
then $2 \mid p^2$,
And $2 \mid p$.
Therefore, $p = 2r$, $r \in \mathbf{N}$
So $2q^2 = (2r)^2 = 4r^2$ and $q^2 = 2r^2$
Then $2 \mid q^2$,
And $2 \mid q$.
Contradiction, since p and q were supposed to be relatively prime
$\therefore$ there does not exist p and q $\in \mathbf{N}$, $(p,q) = 1$ such that $p/q = \sqrt{2}$,
$\therefore \sqrt{2}$ is an irrational number.

Now, I will invite you to linger beside the mathematical action, as I comment on it in the fashion of a Greek chorus, offering asides. I will add the pace, motive, and human texture that transmute the haphazard chorus of events into drama, into story.

We want to prove that $\sqrt{2}$ is irrational. Where to begin? The trouble is that I only know "deficit" definitions of irrationality: the decimals don't end, they don't repeat, it's not rational, it can't be written as p/q. But stories have to be about things that happen, rather than only about thing that *don't* happen, so let me take a slightly different approach and propose the very opposite, that $\sqrt{2}$ it in fact rational. Now what does that tell me about things? Well, if $\sqrt{2}$ is rational, I should be able to write is in a fractional form, as p/q. But if I can write it as p/q, I could also write it as 100p/100q, and a million other ways, so let me honour my characters by introducing them in their barest form, in reduced form. Nobody else can interfere with p/q by dividing them. Anyway, having it in reduced form might come in handy later on!

So I have $p/q = \sqrt{2}$. And I don't like that because it's hard to deal with square roots... It would be so much easier algebraically if the roots were gone, and anyway, I know exactly how to accomplish that, I can write $p^2/q^2 = 2$. But I also don't like working with fractions... I can't get a sense of what that q^2 has to do with anything, so let's write $p^2 = 2\,q^2$. *Aha*! now I'm getting somewhere. This looks nice: nothing is hidden in root signs or in denominators, and p has emerged as the leading actor on the left—the struggle, of p^2/q^2, now has a hero. And what can we say about him, once we've identified him? Well, he is BIGGER than q—fittingly, since he's our hero—and indeed, he's twice as big. So we can see that p^2 is an even number, like 2, 4 or 6. And p is supposed to be a natural number, which means that p^2 actually has to be something like 4, 16, 36. So hey! That means p has to be even too.

So far, so good. Our hero, p, big and natural and even, sure-footed and strong. Now what about that dastardly q? Can I rearrange my equation again? Unmask him in his perfidy? Maybe give q a starring role? Hmmm. Even if I write $q^2 = 2/\,p^2$, it doesn't give me anything useful. How can our hero help? Well! since p is even, he can disguise himself as 2r, where r is some passing natural number. Let's let the algebra do the talking again: I know that $p^2 = 2\,q^2$ which I can write as $(2r)^2 = 2\,q^2$, or $4r^2 = 2\,q^2$, and lo and behold, in a quick battle of numeric soldiers, the 4 smotes the 2 and we are left with $2r^2 = q^2$, and thus, borrowing my logic from before, q being even!!! This is horrible—rather than a hero and a villain, we have two absolute symmetric heroes!!! And that's an impossibility. But now everything has come together. If p and q are both even, p/q can't be in reduced form, which means my assumption at the beginning has to be wrong, which in turn means that $\sqrt{2}$ cannot be a rational number... The emperor has no clothes!!! [...] So $\sqrt{2}$ in this story is sort of like the wizard in the Wizard of Oz... the big, mysterious presence that motivation the actors and their questing, but who in the end is revealed by that action as an imposter. THE END

In addition to being a little more explanatory, or interpretive, this writerly reading of the Pythagorean proof injects emotions and characters. P and q become actors, the number "4" has a role, while the proof by contradiction becomes a sort of romantic maneuver, as Le Lionnais (1948) would call it, one that admits conflict, involves indirectness and perplexes more than enlightens. After all, I never really had to say what an irrational number *is*. And the whole thing seems like a story that can be told again, for other unsuspecting irrational numbers. It becomes a sort of parable for the kinds of things that can happen when you "suppose not!"

While the formal proof, in its terse incarnation, may play the role of a classic text in the mathematics literature, my telling of it was in the present, "lived," intensified by what I know and don't, what I value and don't. It gave opinions, provided questions, united the ideas through drama. Tymoczko (1993) offers a similar narrative of the proof of the Fundamental Theorem of Arithmetic and notes that the narrative is experienced "in a way that one experiences a (short) piece of music or a poem." It exists "in time with a tempo or density, speeding up at certain points and slowing down at others." Once again, time plays a starring role. It attends to its "different rhythms [...], now quickly and easily moving along, now encountering a stumbling block, tension or subsidiary tension." Also the narrative has a certain memorable quality in it. It is coherent. It hangs together. It is easier to remember because the steps have been coloured differently, some highlighted, some forced to the background.

My fellow plenary speaker here, Uri Leron, has—many, many years ago—provided some of the best examples I know of more writerly readings of proofs, in his 1985 FLM article "Heuristic presentations: The role of structuring." Some objected that Uri's proofs were not real proofs because they lacked the presumed linearity of mathematical proofs. Readers of Dickens encountering the "new novel" or recent films such as *Memento* or *Run Lola Run* may have voiced a similar complaint. As always, when artists press on, or expose, our conventions, they are accused of disrespect!

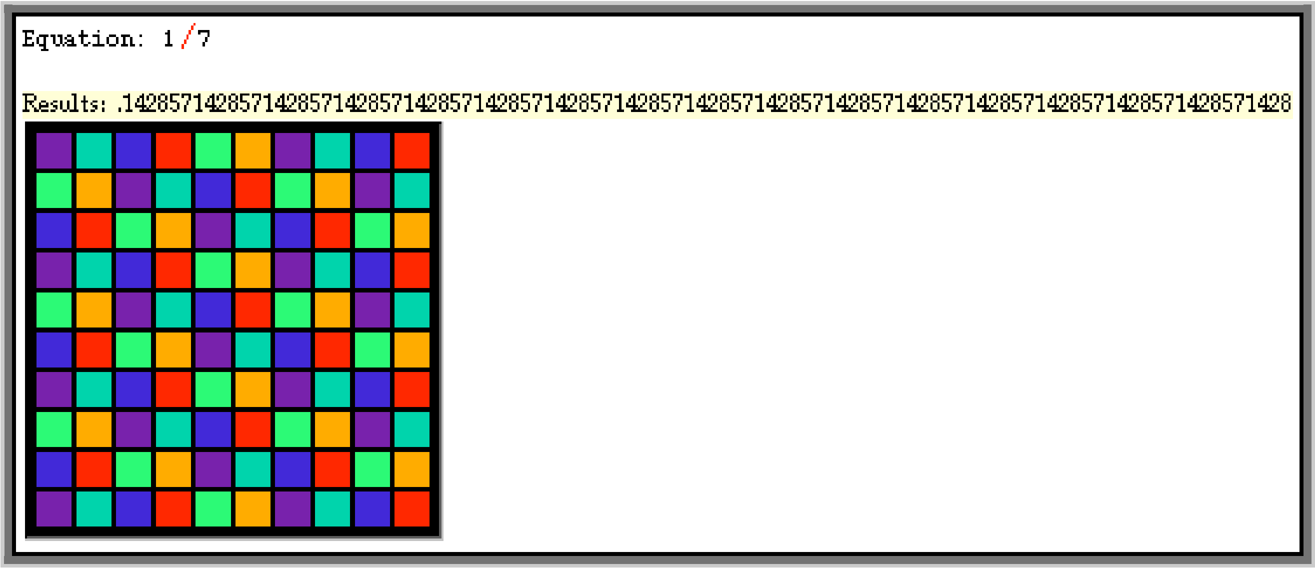
What I have done, in one sense, is read the proof in a writerly way, injecting my own personal and provisional meanings (maybe in a few years, I'll omit some steps, or add some different asides), and ones that run orthogonal to the impetus of the original text. Can we expect kids to do this kind of narration with texts? Probably not, at least not until they grow more aware of the kinds of asides that can live in mathematical texts. So how can we provide them with classroom texts that support and evoke their narrative processes? How do we go about creating materials for students that exemplify this kind of narrative, or "writerly" possibility in mathematics?

Destabilising mathematical texts

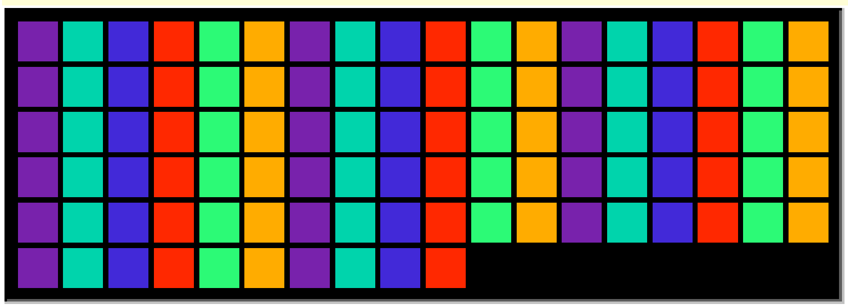
Barthes suggests that destabilizing devices are needed in order to make readerly texts more writerly. He wants to see textual contradictions, ambiguity and gaps. But such words are so loaded in the pedagogical context. Imagine publishers using them as selling points for their textbooks! And even luminaries such as Dewey and Bruner seek to flatten them, pointing out that learning depends on reducing surprise and complexity to simplicity and predictability. But surprise and complexity are exactly the kinds of characteristics cued by Barthes—contradiction, ambiguity, gaps—and operate as preconditions to learning for Dewey and Bruner. So the texts that are apparati of learning contexts have to engage surprise, juxtaposition, multiple meanings and even ambiguity.

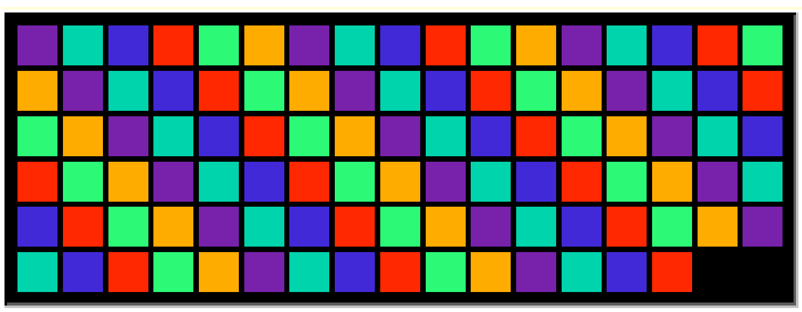
Several years ago my brother Stéfan Sinclair and I created an on-line colour calculator, which I think embodies devices that invite students to become more "writerly" in their mathematical interactions with it, in the sense that it invites them to create their own stories about mathematical ideas and relationships.

The Colour Calculator is an on-line calculator that provides numerical results, but that also offers its results in a colour-coded table. Conventional operations are provided; the division operation allows rational numbers while the square root operator allows irrational numbers. Each digit of the result corresponds to one of ten distinctly coloured swatches in the table. The calculator operates at a maximum precision of 100 decimals digits, and thus it represents each result simultaneously by a (long) decimal string and an array or matrix of colour swatches. It is possible to change the dimension, or the width, of colour table to values between one and 30, as well as the number of decimals that appear. Thus, of particular interest in the Colour Calculator are the pattern-rich real numbers because they can be *seen and understood as patterns of colour.* Rather than emphasising computation and conversion, the CC emphasises the understanding of fractions as patterned objects (as representations) and the relationships between numerators and denominators that determine these patterns (relationships such as prime-, even- and odd-ness). The calculator operations (addition, subtraction, multiplication, division, square root) as well as the changeable width of the table enable exploratory action on the colour patterns themselves.



Using the button that controls the width of the table of colours, one can select different table dimensions that result in different colour patterns, some which highlight interesting aspects of the number's period.





David Hawkins (2000) points out that the mathematics of the Ancient Greeks was often motivated by consideration of both number and shape. However, our contemporary curricula tend to isolate them into different strands and courses. The first thing to notice about the CC is the way in which it returns together, but juxtaposes, numbers (and especially fractions) and shape (in the form of patterns of colours). Now 1/3 *is* "all red" And checkerboards *are* 2/11, 6/11, etc., depending on your colour preferences. Students (as well as mathematicians) are surprised to see numbers this way; in fact, they are destabilized into re-considering their natural, and long-standing aversion to fractions. And I propose that this destabilization permits them to start telling different stories from the usual ones, which often begin with: "I hate fractions."

But the colours and the patterns they make seem also to provide fodder for interpretation, description and personality-building. At an obvious and almost trivial level, every student comments they had never seen fractions or decimals like this, jointly and with colours. Students also note how different this type of mathematical activity is from their regular classroom work, one explaining that "you actually have to *do* things" while another observed that "you have to *notice* things."

These actions and perceptions lead to imaginative interpretations, such as this grade eight student:

> *Okay. Ah. It looks like an abstract painting. Not exactly like a math problem. I'm trying to figure out how it calculates that. Uh. Well, it says that the results are 0.142857 and it repeats. So this is a repeating pattern. I can see it because the red sticks out and the purple, and ooh the green. They kind of go in a diagonal which shows a standard repeating pattern but I'm trying to figure out how things are working. So the number corresponds to the colour…*

Here we see Sean being invited to think in analogies, and to create individual entities with coloured personalities, both which contribute to his process of making sense of what he's seeing and what it means mathematically.

This pre-service elementary teacher who had the opportunity to work with the CC told an even more emotionally charged story about fractions:

> *The repeating fractions have a flow that I find comforting. As a child I disliked fractions that did not terminate, but now I see them in a light of beauty. I find that the decimals which terminate sad. They are unable to touch the fingertips of forever, like the repeating ones can.*

In addition to the strong aesthetic dimension of her experience of fractions, we can see her beginning to let mathematical entities into her life by taking on personalities, I find the development of strong personalities for numbers especially interesting given Keith Devlin's (2000) definition of a mathematician as someone for whom mathematics is a soap opera: "The characters in the mathematical soap opera are not people but mathematical objects—numbers, geometric figures, groups, topological spaces, and so forth. The facts and relationships that are the focus of attention are not births and deaths, marriages, love affairs, and business relationships, but mathematical facts and relationships about mathematical objects". In a similar vein, the calculating wizard Wim Klein remarked "Numbers are friends to me." Taking

3,844 as an example, he said, "For you it's just a three and an eight and a four and a four. But I say 'Hi, 62 squared!'" It seems quite plausible that “colouring” numbers is part of developing personalities for them.

Re-telling stories

I would like to close with one more reflection on the theme of mathematics, stories, learning and computers. Having led dozens of teacher workshops for *The Geometer’s Sketchpad*, I have often been struck by the compulsion these newcomers to Sketchpad have in re-doing what they already know. I have noticed it often, and always marveled, without quite knowing what of make of it. The paper-folding construction of the ellipse is an example. The teachers know what will happen; they know the mathematics; they know the plot and the tricks. But they still want to do it in Sketchpad. Perhaps this is not so different than Gauss’ compulsion to re-prove the law of quadratic reciprocity, over and over again, or Tom Apostol’s motivation to re-prove the irrationality of √2, or even Cézanne’s obsession with painting the Mont Sainte-Victoire.

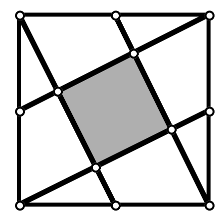
Now of course, Sketchpad’s tools allow you to do many things paper-and-pencil doesn’t. Just as living more experiences allows you to read stories you already know in new ways. So re-doing what you know in Sketchpad may be like re-reading and re-telling yourself stories, including—and this is perhaps much undervalued—the story that you can do mathematics. This phenomenon may not be so different from the pleasure many mathematicians describe of re-creating proofs they have known before. The dramatic tension shifts; it’s no longer about “what’s going to happen”? or “Is it true?” The shift is a meta-cognitive one; you’re interested in how you know what you know. You become part of the story. In a nice self-similarity, the text now includes your previous experience of the text. You anticipate your own past pleasure and become surprised by your own reactions.

In closing...

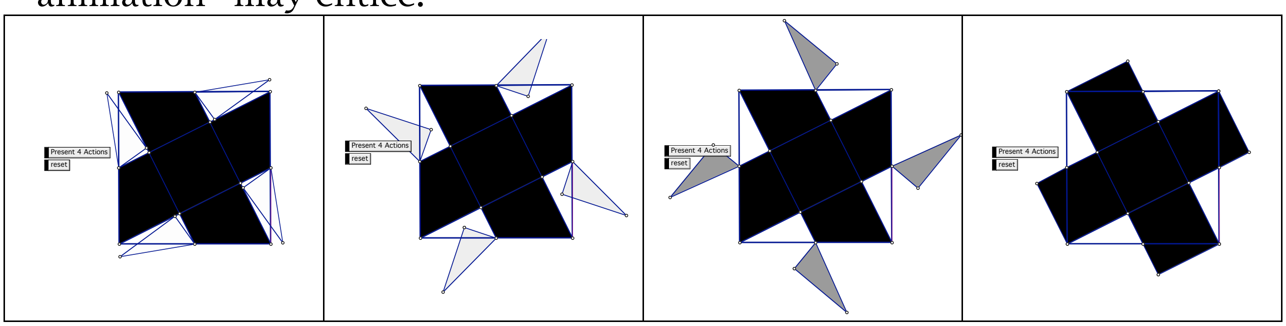
I have showed several examples of educational technologies over the course of this talk, some perhaps more familiar than others. Although the on-line ones I have shown, such as Lulu and the Colour Calculator, have had great success, they possess some basic “writerly” constraints in the sense that the learner (or reader) can never actually create anything new. Sketchpad, on the other hand, and environments such as Logo and Boxer, possesses fewer constraints, and tends to turn students into authors of texts, rather than readers or writers of them. In this example, Kristen, a middle school student, has authored a text on rotational symmetry.

In her sketch, she uses presentation tools such as colour, action buttons, and animation *alongside* the mathematical tools that allow various kinds of transformations, measurements, and geometric constructions. I think these combined tools provide students with the means and impetus to tell a story, much as mathematicians do when they construct proofs. After all, as the mathematician Krull (1987) writes, "mathematicians are not concerned merely with finding and proving theorems; they also want to arrange and assemble the theorems so that they appear not only correct but evident and compelling" (p. 49).

I close with a simple illustration of this kind of story-telling, one in which the presentation tools play less of an extra-mathematical role than they did in the Ferris Wheel. Here, Alex, a grade 8 student, has tried to find a way of arranging and assembling a certain cast of characters to illustrate the following relationship: the large square is 5 times the area of the smaller, tilted square, which is formed by joining the midpoints of the sides of a square to the opposite vertices, as shown in the following figure.



He uses colour, motion, and transformations to illustrate his explanation of the relationship. Given these added features, it is very difficult to communicate the pleasure of this text in paper—though the following "animation" may entice.



I have wandered from hypertext mathematics to poetry and from Greek narratives to Sketchpad animations. The story I wanted to tell this morning will seem to you to have the kinds of gaps and perhaps even contradictions that characterize Barthes' "writerly" texts. I hope that the doors I've left ajar—who can be the narrator in the classroom? Which technologies provide more "writerly" opportunities? What gives a mathematical entity personality?—will provoke discussion and debate during our working sessions. In particular, does the idea of "writerly" texts help us think about how to design classroom texts, be they textbooks, tasks or technologies?

References

Barthes, R. (1975). The Pleasure of the Text. New York: Hill and Wang.

Bruner, J. (1969). *On knowing: Essays for the left hand*. New York: Athaneum.

Devlin, K. (2000). *The math gene: How mathematical thinking evolved and why numbers are like gossip*. New York: Basic Books.

Eco, U. (1994). *Six Walks in the Fictional Woods*. Cambridge, MA: Harvard University Press.

Hawkins, D. (2000). *The roots of literacy*. Boulder, CO: University Press of Colorado.

Krull, W. (1987). The aesthetic viewpoint in mathematics. *The Mathematical Intelligencer, 9*(1), 48-52.

Landow, G. (1992). *Hypertext 2.0: The Convergence of Contemporary Critical Theory and Technology*. London: The John Hopkins University Press.

Le Lionnais, F. (1948/1986). La beauté en mathématiques. In F. Le Lionnais (Ed.), *Les grands courants de la pensée mathématique* (pp. 437-465). Paris: Editions Rivages.

Tymoczko, T. (1993). Value judgements in mathematics: Can we treat mathematics as an art? In A. M. White (Ed.), *Essays in humanistic mathematics* (pp. 67-77). Washington, DC: MAA.