

Children's development of range based estimation skills: Far more than guesswork

The ability to estimate is an essential skill if a person is to feel confident in today's technological and mathematical world. The estimation activities discussed in this proposal help students develop, then refine, their estimates based on authentic contexts and the child's level of mathematical maturity.

The ability to estimate is an essential skill if a person is to feel confident in today's technological and mathematical world. This proposal presents an approach to the development of estimation skills that builds on what Carlow (1986) described as making *intuitive* judgments within a *holistic* framework, and what Taylor-Cox (2001) called *range-based* estimation.

The rationale for finding a range of numbers rather than one definitive number is based on the belief that if an exact answer is required then individuals should be doing something other than estimating, e.g., mental math may be more appropriate. Estimation should be used when it is not important to find an exact answer, or it is too difficult to calculate one quickly. The art (or process) of estimating is often lost when a person believes that the one who guesses closest to the exact number is the best estimator. Early on in our research we noticed that while everybody (adults and children) whom we asked to provide an estimate wanted to offer an exact numerical answer, young children often made very wild guesses. As Leutzinger, Rathmell and Urbatsch explain - - for a young child there is little difference between 35 and 1000, they are both large numbers (1986, p. 82).

Mathematical maturity and the refinement of estimating skills

The skills required by confident estimators need to be included both incidentally and explicitly in the education of young children, and then refined as children's mathematical thinking matures. To this end, a continuum is established, and children progress along this continuum using contexts appropriate to their development. Many writers have discussed the importance of making use of referents to assist students in developing a variety of possible strategies. Carlow (1986) used the term "perceptual anchor", e.g., what might 10, 20, or 100 things look like. Siegel, Goldsmith and Madson (1982) used the term "benchmarks" to describe non-standard units, such as, "How many glasses of water are in a bucket?". The notion of a perceptual anchor, referent, or benchmark is frequently used to visualize how many *chunks* might be in a whole. Reys, Suydam, Lindquist and Smith (1998) refer to the refinement of *chunking* as *unitizing*.

We have introduced estimation activities in Kindergarten by having the teacher model the process of finding the range using a small quantity of concrete materials (between 0 and 30) in an estimation jar. Students show their individual ranges on a number line using a *sticky note* that covers a range of three or four numbers. Placing the sticky note on a number line helps students to visualize the abstract nature of *range*. A discussion of the

strategies and referents used, e.g., “I think a *gummy worm* is about the length of my finger”, helps students reflect upon their estimates and develop constructs for future activities.

As students progress through the grades, the quantities and contexts gradually become more complex. Students begin to learn the process of refining their estimates based on increased knowledge and understanding. One strategy that has proven particularly effective in helping children refine their range is a hundreds chart (see figure 1). The students cross out those numbers on the hundreds chart that they are sure are fewer than their estimate, beginning at the number one. The students are directed to stop when they reach a *maybe* number. Then they cross out the numbers they know for certain are more than their estimate, working backwards from one hundred until they reach another *maybe* number. The range of numbers remaining on the chart becomes their *zone*. Children have the opportunity to start with a fairly large zone that meets their comfort level, and gradually refine that zone as confidence is gained with experience. Those who are risk-takers, or those with advanced mathematical maturity, will attempt to narrow the range more quickly than the hesitant or less mathematically mature child. Everyone can work at his or her own comfort level, providing the teacher with the diagnostic information needed for planning future activities in estimation. Taylor-Cox (2001, p. 214) uses a skillful visualization technique to help students understand the abstract concept of range. She teaches her students to “sandwich” the estimation, and this visualization has been extremely meaningful for young learners.

Following the establishment of their own number zone, students are asked to share their estimation strategies with other class members. The class then counts the number of objects being estimated, and students record whether or not the resulting number is in their zone, or is lower or higher than their zone. Objects are counted in a variety of ways. Placing the objects directly on a number line reinforces one to one correspondence for young learners. Counting out groups of 10 (a group of 10 to each student until only the ones are remaining) not only reinforces place value for older students, it demonstrates what a *chunk* of 10 looks like for future reference. Groups of 100 would effectively demonstrate larger quantities. Students are encouraged to reflect on the strategies they personally used when estimating, record these in the section titled “reflections”, then consider what strategies might be utilized in the future (see figure 2). Again, students are encouraged to discuss their strategies with their peers. It is important for them to understand that a variety of strategies can be used for the same estimation activity. Sharing ideas encourages diversity for future occasions. The goal is to promote mathematical thinking and worthwhile discussion, not to ascertain who has the correct answer. Students are invited to bring objects from home each week to further encourage real life estimation activities. This extension to the home increases parental involvement and facilitates family participation in rich mathematical experiences using every day materials.

Establishing a positive environment

It should be noted that in order to provide a safe environment where risk-taking is encouraged, we recommend what Chester Carlow (1986) referred to as a “light touch” when introducing estimating. Students are often reluctant to take risks, wanting to have the *correct* answer. It is crucial for them to appreciate the value of *not knowing*, so that they can use these occasions as opportunities for growth rather than anxiety. Teachers should model risk-taking by conveying their own comfort level in not knowing an exact answer. The emphasis can then be placed upon the strategies used to find a reasonable range, which in turn leads to a deeper understanding of the mathematical concepts involved. Estimation can help students recognize the benefits associated with taking risks in mathematics. Gradually, they begin to enjoy the challenges expected of them and have the confidence to apply their estimating skills in a variety of problem solving contexts.

Discussion

Children develop a strong sense of number and measurement when they have real world referents to visualize mathematical abstractions. Estimation provides a starting point for constructing some of these early reference points. Developing a comfort level with *holistic* or *range-based* estimation requires a variety of experiences in many different contexts. The types of estimation activities discussed in this proposal help students of varying ages develop a sense of number, quantity and/or size. Discussing with children and parents how they use estimation in their everyday lives (how much they are spending at the grocery store, how far they can go without stopping for gas, how much spaghetti will feed the family) will help them to see that estimation is a skill that we use and depend upon without always realizing its importance. The skill of estimating is developed through practice and reflection. The more individuals estimate, and the more they think about how they do it, the better they become. To be confident and successful estimators, children need a variety of opportunities to practice and to learn from their experiences. Estimating should be an ongoing activity, from its introduction in kindergarten to the more sophisticated processes of later grades. A well-planned program includes a selection of estimating activities for different levels of mathematical maturity, using contexts such as time, distance, capacity and area.

References

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Learning to Estimate



Are you **“IN THE ZONE?”**

Draw or describe what is in the Estimating Jar.

Instructions:

- Start at 1. Cross out the numbers on the 100-chart that you think are definitely NOT the number of objects in the estimating jar. STOP when you reach a “maybe” number.
- Work backwards from 100. Cross out the numbers that you think are definitely NOT the number of objects in the estimating jar. STOP when you reach a “maybe” number.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The numbers left are **“The Zone”** or the range of reasonable guesses. My

“zone” is more than _____ but less than _____.

The Count



Count the items in the estimating jar. Record the count below.

We counted _____ .
(Number) (Items we counted in the jar.)

The number counted was:

- In the zone.
- Less than the zone.
- Greater than the zone.

Reflections

- What strategies helped you to be “in the zone”?
- What strategies could you use next time?

Explain your thinking.

Remember:

The more you estimate the better you become at finding **“the zone”!**

(L. Adams/Esso Family Math)